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# Open Superstring and Noncommutative Geometry

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## Abstract

We perform canonical quantization of the open Neveu-Schwarz-Ramond (NSR) superstrings in the background of a  $D$ -brane with the NS B-field. If we choose the mixed boundary condition as a primary constraint, it generates a set of secondary constraints. These constraints are easily solved and as a result, the noncommutative geometry in the bosonic string theory is extended to the superspace. Solving the constraint conditions we also find that the Hamiltonian for the superstring is equivalent to a free superstring Hamiltonian on the target space with the effective open string metric  $G$ .

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## I. INTRODUCTION

The noncommutative geometry [1] has been considered for some time in connection with various physics subjects, which include the lowest Landau level physics in condensed matter physics, the quantum plane in mathematical physics, and the geometrical interpretation of Yang-Mills-Higgs action and so forth. Recent motivation to study the noncommutative geometry mainly comes from the string theory. If the matrix model of M-theory [2] is compactified on tori in the presence of an appropriate background field, the noncommutative supersymmetric Yang-Mills theory arises [3]. It implies that the  $D$ -brane world volume theory is noncommutative. This point has been discussed later by a more direct approach. If we quantize the open string in the background of a  $D$ -brane with the NS  $B$ -field, the ends points of the string, attached on the  $D$ -brane is shown to be noncommutative [4,5]. The effective action for the  $D$ -brane in the low energy regime is induced by the open string on it, thus it should be noncommutative. In their recent paper [6] Seiberg and Witten extensively discussed the various aspects of the noncommutative geometry in the context of the string theory such as the equivalence between ordinary gauge fields and the noncommutative gauge fields, Morita equivalence, and its implications in M-theory.

The noncommutativity in the open string theory can be most easily seen in the framework of canonical quantization. The approach based on the canonical quantization was adopted in the earlier works [5] on the open string in the  $D$ -brane background and further elaborated recently in refs. [7,8]. In [7] we show that the set of constraints generated by the mixed boundary condition can be explicitly solved. Solving the constraint conditions we find the open string is governed by a free string Hamiltonian defined on the target space with the effective metric  $G$ . Resorting to the canonical analysis we also evaluate the Polyakov path integral for the open string and obtain the noncommutative Dirac-Born-Infeld action as the low energy effective action for the  $D$ -brane, which reduces to the noncommutative Yang-Mills theory in the zero slope limit. If the constraint conditions are imposed, the Wilson loop operator with the ordinary gauge field becomes the Wilson loop operator with

a noncommutative gauge field. Hence the Seiberg-Witten map, which connects the ordinary gauge field to the noncommutative one, can be understood in the framework of the canonical quantization [8].

The appearance of the noncommutative geometry in the bosonic string theory is now well understood. However, the noncommutative geometry in the framework of the superstring theory needs further study. Since the  $D$ -brane is a BPS object, many interesting features of the  $D$ -brane are associated with the supersymmetry. Therefore, it is important to understand how the noncommutativity interplay with the supersymmetry. In the present paper we attempt to extend the previous canonical analysis to the supersymmetric theory. To this end we take the Neveu-Schwarz-Ramond (NSR) superstring [9] in the background of  $D$ -brane with NS  $B$ -field.

## II. THE NSR SUPERSTRING IN THE BACKGROUND OF $D$ -BRANE

We begin with the world sheet action for the free NSR superstring in superspace. In the presence of the  $D$ -p-brane the action for the superstring is given as

$$S = \frac{1}{4\pi\alpha'} \int_M d^2\xi d^2\theta g_{\mu\nu} D_+ Z^\mu D_- Z^\nu \quad (1)$$

$$Z^\mu(\xi, \theta_\pm) = X^\mu + \theta_+ \psi_-^\mu + \theta_- \psi_+^\mu + \theta_+ \theta_- F^\mu$$

$$D_+ = \frac{\partial}{\partial \theta_-} - i\theta_- \partial_+, \quad D_- = -\frac{\partial}{\partial \theta_+} + i\theta_+ \partial_-$$

where we choose the world sheet metric as  $(-, +)$  and  $F^\mu$  is the auxiliary field.  $Z^a$ ,  $a = p+1, \dots, 9$  of the transverse directions are subject to the Dirichlet boundary condition,  $\partial_\tau Z^a = 0$ . If the  $D$  brane carries the NS  $B$ -field, the action for the longitudinal coordinates  $Z^i$ ,  $i = 0, \dots, p$ , is replaced with the following one

$$S = \frac{1}{4\pi\alpha'} \int_M d^2\xi d^2\theta (g + 2\pi\alpha' B)_{ij} D_+ Z^i D_- Z^j. \quad (2)$$

Since the action for the transverse coordinates is rather trivial, we will be concerned with the action for the longitudinal coordinates only hereafter. For the background with constant  $g$  and  $B$ , the action is greatly simplified to

$$S = \frac{1}{4\pi\alpha'} \int_M d^2\xi \left( E_{ij} \partial_+ X^i \partial_- X^j - i\psi_+^i E_{ij} \partial_- \psi_+^j - i\psi_-^i E_{ij}^T \partial_+ \psi_-^j \right), \quad (3)$$

$$E_{ij} = (g + 2\pi\alpha' B)_{ij}.$$

The boundary conditions are given as

$$g_{ij} \partial_\sigma X^i - 2\pi\alpha' B_{ij} \partial_\tau X^i = 0, \quad (4a)$$

$$E_{ij} \psi_+^j - E_{ij}^T \psi_-^j = 0, \quad (4b)$$

for  $\sigma = 0, \pi$ .

For canonical quantization of the bosonic part we refer to ref. [7]: The boundary condition Eq.(4a) generates a set of second class constraints, which can be explicitly solved. After solving the constraints, the bosonic part of the Hamiltonian and the string coordinate variables are obtained as

$$H_B = (2\pi\alpha') \frac{1}{2} p_i (G^{-1})^{ij} p_j + (2\pi\alpha') \sum_{n=1} \left\{ \frac{1}{2} K_{in} (G^{-1})^{ij} K_{jn} + \frac{1}{(2\pi\alpha')^2} \frac{n^2}{2} Y_n^i G_{ij} Y_n^j \right\}, \quad (5a)$$

$$X^i(\sigma) = x^i + \theta_{NC}^{ij} p_j \left( \sigma - \frac{\pi}{2} \right) + \sqrt{2} \sum_{n=1} \left( Y_n^i \cos n\sigma + \frac{1}{n} \theta_{NC}^{ij} K_{jn} \sin n\sigma \right) \quad (5b)$$

where  $Y_n^i$  and  $K_n^i$  satisfy the usual commutation relation

$$[Y_n^i, Y_m^j] = 0, \quad [Y_n^i, K_{jm}] = i\delta^i_j \delta_{nm}, \quad [K_{in}, K_{jm}] = 0 \quad (6)$$

and

$$\theta_{NC}^{ij} = -(2\pi\alpha')^2 \left( \frac{1}{g + 2\pi\alpha' B} B \frac{1}{g - 2\pi\alpha' B} \right)^{ij} \quad (7a)$$

$$(G^{-1})^{ij} = \left( \frac{1}{g + 2\pi\alpha' B} g \frac{1}{g - 2\pi\alpha' B} \right)^{ij}. \quad (7b)$$

Now let us turn to the fermionic part. The NSR string has two sectors, depending on the boundary conditions for the fermionic variables: The Ramond sector with periodic boundary condition

$$\psi^{i+}(\sigma) = \sum_n \psi_n^{i+} e^{in\sigma}, \quad \psi^{i-}(\sigma) = \sum_n \psi_n^{i-} e^{-in\sigma}, \quad (8)$$

and the Neveu-Schwarz sector with anti-periodic boundary condition

$$\psi^{i+}(\sigma) = \sum_n \psi_{n+1/2}^{i+} e^{i(n+1/2)\sigma}, \quad \psi^{i-}(\sigma) = \sum_n \psi_{n+1/2}^{i-} e^{-i(n+1/2)\sigma} \quad (9)$$

where  $n \in \mathbf{Z}$ . Since the constraint Eq.(4b) is linear in the fermionic variables, it is compatible with these boundary conditions.

We discuss the Ramond sector first. The canonical analysis of the Neveu-Schwarz sector is not much different from that of the Ramond sector. In the Ramond sector the fermionic part of the action in the normal modes is written as

$$L_F = -i \sum_n (\psi_n^{i+} g_{ij} \dot{\psi}_{-n}^{j+} + \psi_n^{i-} g_{ij} \dot{\psi}_{-n}^{j-}) - H, \quad (10a)$$

$$H_F = \sum_n n (\psi_n^{i+} E_{ij} \psi_{-n}^{j+} + \psi_n^{i-} E_{ij}^T \psi_{-n}^{j-}). \quad (10b)$$

We can get the Poisson bracket from the this canonical form as

$$\{\psi_n^{i+}, \psi_m^{j+}\}_{PB} = i(g^{-1})^{ij} \delta(n+m), \quad \{\psi_n^{i-}, \psi_m^{j-}\}_{PB} = i(g^{-1})^{ij} \delta(n+m). \quad (11)$$

The boundary conditions accordingly read as

$$\varphi_{0i} = E_{ij} \sum_n \psi_n^{j+} - E_{ij}^T \sum_n \psi_n^{j-} = 0, \quad (12a)$$

$$\bar{\varphi}_{0i} = E_{ij} \sum_n (-1)^n \psi_n^{j+} - E_{ij}^T \sum_n (-1)^n \psi_n^{j-} = 0. \quad (12b)$$

We may choose the boundary condition Eq.(12a) as a primary constraint. Then the Dirac procedure requires to introduce the following secondary constraint in order to be consistent

$$[H, \varphi_i]_{PB} = i \sum_n n (E_{ij} \psi_n^{j+} - E_{ij}^T \psi_n^{j-}) = 0. \quad (13)$$

It yields the secondary constraint as

$$\varphi_{1i} = \sum_n n (E_{ij} \psi_n^{j+} - E_{ij}^T \psi_n^{j-}) = 0. \quad (14)$$

Then the Dirac procedure further requires

$$[H, \varphi_{1i}]_{PB} = 0 \quad (15)$$

It leads us to another secondary constraint

$$\varphi_{2i} = \sum_n n^2 (E_{ij}\psi_n^{j+} - E_{ij}^T\psi_n^{j-}) = 0. \quad (16)$$

We may repeat this procedure until no new secondary constraints are generated. By repetition we get

$$\varphi_{mi} = \sum_n n^m (E_{ij}\psi_n^{j+} - E_{ij}^T\psi_n^{j-}) = 0, \quad m = 1, 2, \dots \quad (17)$$

Since the obtained constraints are of second class, we should introduce the Dirac bracket. However, it may not be convenient to construct the Dirac bracket with this set of constraints,  $\{\varphi_{mi} = 0, m = 1, 2, 3, \dots\}$ . As in the case of the bosonic string theory, the following observation turns out to be very useful: We can easily disentangle the set of constraints and find that they are equivalent to

$$\{E_{ij}\psi_n^{j+} - E_{ij}^T\psi_n^{j-} = 0, \quad n \in \mathbf{Z}\}. \quad (18)$$

(We may also take the boundary condition, Eq.(12b), imposed on the other end of the open superstring. But it generates the same set of the constraints, Eq.(18); it is redundant.)

The fermionic degrees of freedom are halved by the set of conditions Eq.(18). They reduce to

$$\{\psi_n^{i+} - \psi_n^{i-} = 0, \quad n \in \mathbf{Z}\} \quad (19)$$

when the NS  $B$ -field is absent. In the case of the free superstring theory we get rid of  $\psi_n^{i-}$  in favor of  $\psi_n^{i+}$  and choose  $\{\psi_n^{i+}\}$  as a proper basis for the fermionic degrees of freedom. Suppose that we choose  $\{\psi_n^{i+}\}$  as the basis for the fermionic degrees of freedom in the present case. If we make use of the constraints,  $\psi_n^{i-} = ((E^T)^{-1}E)^i{}_j\psi_n^{j+}$ , we find that the fermionic part of the Lagrangian is written as

$$L_F = -i \sum_n (\psi_n^{i+} g_{ij} \dot{\psi}_{-n}^{j+}) - H_F, \quad (20)$$

$$H_F = \sum_n n (\psi_n^{i+} g_{ij} \psi_{-n}^{j+})$$

where we make use of

$$E^T E^{-1} g (E^{-1})^T E = E^T G^{-1} E = g. \quad (21)$$

Here  $\psi^{i+}$  is scaled as  $\psi^{i+} \rightarrow \psi^{i+}/\sqrt{2}$ . At first appearance the Lagrangian looks same as that of the free superstring and the NS  $B$ -field does not affect the fermionic part. However, this conclusion is misleading. If the constraint conditions are imposed the bosonic part respects the effective metric  $G$  instead of  $g$ . If the fermionic part still respects the metric  $g$  after imposing the constraints, the super-Virasoro algebra would not be consistent. We will discuss this point in the next section in some detail.

### III. SUPERSYMMETRY AND NONCOMMUTATIVE GEOMETRY

Let us recall the bosonic part of the Hamiltonian for the open string in the background of NS  $B$ -field. If the background NS  $B$ -field is constant, the Hamiltonian is given as

$$H^B = \frac{1}{2} (L_0^B + \bar{L}_0^B), \quad (22a)$$

$$L_0^B = (2\pi\alpha') p_{Li} (g^{-1})^{ij} p_{Lj} + (2\pi\alpha') \sum_{n=1} n A_{in} (g^{-1})^{ij} A_{jn}^\dagger, \quad (22b)$$

$$\bar{L}_0^B = (2\pi\alpha') \frac{1}{2} p_{Ri} (g^{-1})^{ij} p_{Rj} + (2\pi\alpha') \sum_{n=1} n \bar{A}_{in} (g^{-1})^{ij} \bar{A}_{jn}^\dagger, \quad (22c)$$

where

$$p_{Li} = \frac{1}{\sqrt{2}} \left( p_i - \frac{1}{2\pi\alpha'} E_{ij}^T a^j \right), \quad (23a)$$

$$p_{Ri} = \frac{1}{\sqrt{2}} \left( p_i + \frac{1}{2\pi\alpha'} E_{ij} a^j \right). \quad (23b)$$

The left and right movers are defined as

$$A_{in}^\dagger = \frac{1}{\sqrt{2n}} \left( P_{i-n} + \frac{in}{2\pi\alpha'} E_{ij}^T X_n^j \right), \quad (24a)$$

$$A_{in} = \frac{1}{\sqrt{2n}} \left( P_{in} - \frac{in}{2\pi\alpha'} E_{ij}^T X_{-n}^j \right), \quad (24b)$$

$$\bar{A}_{in}^\dagger = \frac{1}{\sqrt{2n}} \left( P_{in} + \frac{in}{2\pi\alpha'} E_{ij} X_{-n}^j \right), \quad (24c)$$

$$\bar{A}_{in} = \frac{1}{\sqrt{2n}} \left( P_{i-n} - \frac{in}{2\pi\alpha'} E_{ij} X_n^j \right), \quad (24d)$$

and their commutation relations are

$$[A_{in}, A_{jm}^\dagger] = (2\pi\alpha')^{-1} g_{ij} \delta_{nm}, \quad [\bar{A}_{in}, \bar{A}_{jm}^\dagger] = (2\pi\alpha')^{-1} g_{ij} \delta_{nm}. \quad (25)$$

Using the constraint conditions

$$a^i = \theta^{ij} p_j, \quad \bar{Y}_n^i = \frac{1}{\sqrt{2}} (X_n^i - X_{-n}^i) = \frac{1}{n} \theta^{ij} K_{jn}, \quad \bar{K}_{in} = \frac{1}{\sqrt{2}} (P_n^i - P_{-n}^i) = 0, \quad (26)$$

we may remove  $a^i$ ,  $\bar{Y}_n^i$ ,  $\bar{K}_{in}$  in favor of  $(x^i, p_i)$ ,  $(Y_n^i, K_{in})$ , which are canonical variables for the open string. As a result we find that the bosonic part of the Hamiltonian is just the same as the free Hamiltonian for the open string on the target space with metric  $G$

$$H^B = (2\pi\alpha') \frac{1}{2} \left( p^2 + \sum_{n=1} n A_{in}(E') \left( G^{-1} \right)^{ij} A_{jn}^\dagger(E') \right), \quad (27a)$$

$$A_{in}(E') = \frac{1}{\sqrt{2n}} \left( K_{in} - \frac{in}{2\pi\alpha'} G_{ij} Y_n^j \right), \quad (27b)$$

$$A_{in}^\dagger(E') = \frac{1}{\sqrt{2n}} \left( K_{in} + \frac{in}{2\pi\alpha'} G_{ij} Y_n^j \right), \quad (27c)$$

where the left movers and the right movers satisfy

$$[A_{in}(E'), A_{jm}^\dagger(E')] = (2\pi\alpha')^{-1} G_{ij} \delta(n+m). \quad (28)$$

It is interesting to note [10] that the left and right movers  $A^\dagger(E)$ ,  $\bar{A}^\dagger(E)$  are related to the left and right movers,  $A^\dagger(E')$ ,  $\bar{A}^\dagger(E')$  by a T-dual transformation [11]

$$T = \begin{pmatrix} I & 0 \\ (2\pi\alpha')^{-1} \theta & I \end{pmatrix}. \quad (29)$$

We may have geometric interpretation of this T-dual transformation as discussed in ref. [12].

Now let us turn to the fermionic constraint  $F_0$ , which forms the super-algebra with the Hamiltonian  $H$ ,

$$\{F_0, F_0\} + \{\bar{F}_0, \bar{F}_0\} = 2 \left( L_0 + L_0^F \right) + 2 \left( \bar{L}_0 + \bar{L}_0^F \right) = 2 \left( H^B + H^F \right). \quad (30)$$

In the presence of the NS  $B$ -field the fermionic constraint  $F_0$  is given as

$$(2\pi\alpha')^{-\frac{1}{2}}F_0 = p_{Li}\psi_0^{i+} + \sum_{n=1} \sqrt{n} \left( A_{in}\psi_{-n}^{i+} + A_{in}^\dagger\psi_n^{i+} \right), \quad (31a)$$

$$(2\pi\alpha')^{-\frac{1}{2}}\bar{F}_0 = p_{Ri}\psi_0^{i-} + \sum_{n=1} \sqrt{n} \left( \bar{A}_{in}\psi_{-n}^{i-} + \bar{A}_{in}^\dagger\psi_n^{i-} \right). \quad (31b)$$

From the canonical analysis of the bosonic part we expect that the fermionic constraint is rewritten as

$$(2\pi\alpha')^{-\frac{1}{2}} \left( F_0 + \bar{F}_0 \right) = p_i \hat{\psi}^i + \sum_{n=1} \sqrt{n} \left( A_{in}^\dagger(E')\hat{\psi}_n^i + A_{in}(E')\hat{\psi}_{-n}^i \right) \quad (32)$$

where the fermion operator  $\hat{\psi}_n^i$  satisfies

$$\{\hat{\psi}_n^i, \hat{\psi}_m^j\}_{PB} = (G^{-1})^{ij}\delta(n+m). \quad (33)$$

It follows then that the fermion operator  $\hat{\psi}_n^i$  may be defined as

$$\hat{\psi}_n^i = \frac{1}{\sqrt{2}} \left( \psi_n^{i+} + \psi_n^{i-} \right) = \sqrt{2}(G^{-1}E)^i{}_j \psi^{j+} = \sqrt{2}(G^{-1}E^T)^i{}_j \psi^{j-}. \quad (34)$$

Rewriting the fermionic part of the Lagrangian Eq.(10) as

$$L_F = -i \sum_n \left( \hat{\psi}_n^i G_{ij} \partial_\tau \hat{\psi}_{-n}^j \right) - H_F, \quad (35a)$$

$$H_F = \sum_n n \left( \hat{\psi}_n^i G_{ij} \hat{\psi}_{-n}^j \right), \quad (35b)$$

we confirm that the fermionic part also respects the effective open string metric  $G$ . It is consistent with the commutation relation among  $\{\hat{\psi}_n^i\}$  and the supersymmetry.

Being equipped with the canonical analysis of the fermionic part, we discuss the non-commutativity in the superspace

$$Z^i(\sigma) = X^i(\sigma) + \theta_+ \sum_n \psi_n^{i-} e^{-in\sigma} + \theta_- \sum_n \psi_n^{i+} e^{in\sigma} + \theta_+ \theta_- \sum_n F^i e^{in\sigma}. \quad (36)$$

We may rewrite the fermionic part of  $Z^i$  in terms of  $\hat{\psi}_n^i$  as

$$Z_F^i = \frac{1}{\sqrt{2}} \sum_n \left\{ \left( \theta \cos n\sigma - i\bar{\theta} \sin n\sigma \right) \hat{\psi}_n^i + i \frac{(\theta_{NC}G)^i{}_j}{2\pi\alpha'} \left( \theta \sin n\sigma + i\bar{\theta} \cos n\sigma \right) \hat{\psi}_n^j \right\} \quad (37)$$

where

$$\theta = \frac{1}{\sqrt{2}}(\theta_+ + \theta_-), \quad \bar{\theta} = \frac{1}{\sqrt{2}}(\theta_+ - \theta_-), \quad \bar{\theta}\theta = \theta_+\theta_-.$$

Note that the end points of the string are no longer holomorphic in  $\theta$

$$Z_F^i(0) = \frac{1}{\sqrt{2}} \sum_n \left( \theta \hat{\psi}_n^i - \bar{\theta} \frac{(\theta_{NC} G)^i{}_j}{2\pi\alpha'} \hat{\psi}_n^j \right), \quad (38a)$$

$$Z_F^i(\pi) = \frac{1}{\sqrt{2}} \sum_n (-1)^n \left( \theta \hat{\psi}_n^i - \bar{\theta} \frac{(\theta_{NC} G)^i{}_j}{2\pi\alpha'} \hat{\psi}_n^j \right). \quad (38b)$$

It may have some consequences in construction of the vertex operators. The noncommutativity can be easily seen if we evaluate the commutator between  $Z_F^i$

$$[Z^i(\sigma), Z^j(\sigma')] = [X^i(\sigma), X^j(\sigma')] + [Z_F^i(\sigma), Z_F^j(\sigma')]. \quad (39)$$

The noncommutativity in the bosonic sector is discussed in details in the previous works [5,7]. The commutation relation between  $Z_F^i(\sigma)$  is found to be

$$[Z_F^i(\sigma), Z_F^j(\sigma')] = \begin{cases} \frac{i}{2\pi\alpha'} (\theta\bar{\theta}) \theta_{NC}^{ij} & : \sigma = \sigma' = 0, \\ -\frac{i}{2\pi\alpha'} (\bar{\theta}\theta) \theta_{NC}^{ij} & : \sigma = \sigma' = \pi \\ 0 & : \text{otherwise} \end{cases} \quad (40)$$

where we use  $\zeta(0) = \sum_n 1 = -1/2$ .

The canonical analysis of the Neveu-Schwarz Sector is obtained as we replace the integer modes of the fermion variables by the half-integer modes,  $\psi_n \rightarrow \psi_{n+1/2}$ .

#### IV. CONCLUDING REMARKS

A few remarks are in order to conclude the paper. The first one is a brief summary of the present work. We perform the canonical quantization of the NSR open superstring attached on the  $D$ -brane with a NS  $B$ -field. The open superstring has fermionic boundary conditions to be imposed on the ends of the string in addition to the bosonic ones. Taking the fermionic boundary conditions as primary ones, we obtain a set of infinite secondary constraints, which turn out easy to solve. Choosing an appropriate basis for the fermion variables, we find that the fermion variables also respect the effective metric  $G$  and the fermionic part of the Hamiltonian is just same as that of the free open string Hamiltonian

defined on the target space with the effective metric  $G$ . Thus, the canonical analysis of the bosonic part is extended to the fermionic part explicitly. The interesting point we should note is that the superspace coordinate  $Z^i$  is no longer holomorphic in  $\theta$  at the end points of the string. We expect that this has some important consequences when we construct the vertex operators representing physical states.

The present canonical analysis suggests a number of interesting directions to explore along the line of this work. We may construct the vertex operator for emission of a scalar to study the recent issues associated with the noncommutative field theories [13] in the context of superstring theory. The supersymmetric Dirac-Born-Infeld action [14] may be derived if we construct the vertex operator for emission of a massless vector and evaluate the Polyakov string path integral over a disk on the D-brane world sheet. It would also bring us to the Seiberg-Witten map in the context of the supersymmetric noncommutative field theory. The T-duality also deserves further study and the Morita equivalence may be extended to the supersymmetric theory. After all these related subjects may be discussed in a single framework of the second quantized open superstring theory, if properly constructed. The canonical analysis presented here would be certainly useful to develop the second quantized open superstring theory on the noncommutative geometry.

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